Regression Analysis (II)
Final Exam.

Dec. 18, 2017

1.(120) Consider a logistic regression model $logit(\pi) = \beta_0 + \beta_1 X$, and assume that the response $Y_i \sim B(m_i, \pi_i),\ i = 1, 2, \cdots, n$.

(1)(30) Show that the natural link function is logit.

(2)(30) Compute the log-likelihood function l in terms of β_0 and β_1.

(3)(30) Show that $\frac{\partial^2 l}{\partial \beta_1^2} = \sum_{i=1}^n m_i x_i^2 \pi_i (1 - \pi_i)$.

(4)(30) Compute the Pearson residual.

2.(150) Consider a oneway ANOVA model $Y_{jk} = \mu + \alpha_j + \epsilon_{jk}\ , j = 1, 2;\ k = 1, 2, 3$, where ϵ is iid $N(0, 1)$. Data set is given as follows;

$trt\ A : 4, 6, 5; \quad trt\ B : 5, 4, 3$

(1)(30) When we write the model in matrix form $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$, express $\mathbf{X}$, $\boldsymbol{\beta}$ and $\mathbf{y}$.

(2)(30) Show that $\mathbf{X}^t\mathbf{X}$ is singular, and compute $\boldsymbol{\beta}$ using the corner point restriction.

(3)(30) Compute the deviance D_1.

(4)(30) Compute the deviance D_0 under $H_0 : \alpha_1 = \alpha_2 = 0$.

(5)(30) Derive the chi-square test statistic and the $F-$statistic to test $H_0 : \alpha_1 = \alpha_2 = 0$.

3.(30) Assume that the response Y is a discrete r.v. with k categories, and $\mathbf{x}$ is a $p-$dimensional covariate. Construct a proportional odds model, and explain the model in detail.

4.(60) Consider a multiple linear regression model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$.

(1)(30) Assume that $\boldsymbol{\epsilon} \sim N_n(\mathbf{0}, \sigma^{\mathbf{2}}\mathbf{I})$, and that the prior for $\boldsymbol{\beta}$ is $N_p(\mathbf{0}, \sigma^{\mathbf{2}}\mathbf{V})$. Find the variance-covariance matrix posterior distribution for $\boldsymbol{\beta}$.

(2)(30) Find the Bayes estimator for $\boldsymbol{\beta}$ under the squared error loss.

5.(40) Assume that $\mathbf{z} \sim N_p(\mathbf{0}, \theta\mathbf{I})$, and let $V = \mathbf{z}^t\mathbf{z}/\theta$. Compute $E(V^{-1})$